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ON THE VIABILITY OF GRAVITATIONAL BOSE-EINSTEIN CONDENSATES AS ALTERNATIVES TO SUPERMASSIVE BLACK HOLES

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ABSTRACT

Black holes are inevitable mathematical outcome of spacetime-energy coupling in general relativity. Currently these objects are of vital importance for understanding numerous phenomena in astrophysics and cosmology.

However, neither theory nor observations have been capable of unequivocally prove the existence of black holes or granting us an insight of what their internal structures could look like, therefore leaving researchers to speculate about their nature.

In this paper the reliability of supermassive Bose-Einstein condensates (henceforth SMBECs) as alternative to supermassive black holes is examined. Such condensates are found to suffer of a causality problem that terminates their cosmological growth toward acquiring masses typical for quasars and triggers their self-collapse into supermassive black holes (SMBHs).

It is argued that a SMBEC-core most likely would be subject to an extensive deceleration of its rotational frequency as well as to vortex-dissipation induced by the magnetic fields that thread the crust, hence diminishing the superfluidity of the core. Given that rotating superfluids between two concentric spheres have been verified to be dynamically unstable to non axi-symmetric perturbations, we conclude that the remnant energy stored in the core would be sufficiently large to turn the flow turbulent and dissipative and subsequently lead to core collapse bosonova.

The feasibility of a conversion mechanism of normal matter into bosonic condensates under normal astrophysical conditions is discussed as well.

We finally conclude that in lack of a profound theory for quantum gravity, BHs should continue to be the favorite proposal for BH candidates.

Subject headings: Relativity: general, neutron stars, black hole, dark objects, boson objects — condensed matter physics: superfluids, Bose-Einstein condensate — cosmology: dark energy — fluids: superfluids, superconduction

1. INTRODUCTION

Before 227 years ago and 118 after Newton's gravitation (1666), John Michell (1787) was the first to anticipate the existence of a critical radius, R_H, below which even the light cannot escape the gravitational pull of the central point mass object. Based on Newtonian gravity, he equated the potential energy E_p of the object of mass M to the kinetic energy E_K and obtained: $R_{crit} = 2GM/c^2$, where "c, G" are the speed of light and the gravitational constant, though the speed of light was still uncertain and the possible existence of such light-capturing objects in nature was merely a fictitious proposal.

In 1915 presented Einstein his profound theory of gravitation, therein postulating the energy as a field in four-dimensional spacetime and posting it as source for curvature. Mathematically, Einstein field equations read:

$$\mathrm{G}_{\mu\nu} = \kappa \mathrm{T}_{\mu\nu}, \tag{1}$$

where $\mathrm{G}_{\mu\nu}, \mathrm{T}_{\mu\nu}$ are the 2-rank Einsteins's geometrical and energy-momentum tensors, respectively. The coefficient $\kappa = 8\pi \mathrm{G}/\mathrm{c}^2 \approx 1.863 \times 10^{-27}$.

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Obviously, unless there is a unusually strong accumulation of energy in a relatively small volume, the LHS and the RHS of Eq. (1) can be conveniently decoupled, therefore justifying the Newtonian theory in the weak field limit in flat spacetime.

Shortly thereafter the GR was published in (1915), Karl Schwarzschild presented a solution for the field equations that corresponds to a spherically symmetric object of mass M embedded in vacuum. This solution was the first theoretical proof that BHs are inevitable products of general relativity (GR), where the spacetime becomes indefinitely warped.

Several years later, Subrahmanyan Chandrasekar (1931) proposed a framework for the BH-creation. Accordingly, sufficiently massive stars could in principle collapse under their own self-gravity, as neither the thermal nor the degenerate pressures could counter-balance the enormous gravitational attraction.

From the point of view of Schwarzschild's solution, a massive star could contract in a quasi-stationary manner to finally undergo a dynamical collapse into indefinitely small size at the center. However, a sufficiently distant observer would be able to observe the collapsing matter up to the horizon, but not beyond.

In the early times, this scenario was rejected both by theorists and observers alike and in particular by Einstein and Eddington. Only at the beginning of the sixties the BH-proposal was revived, when BHs appeared to be the only reasonable objects to explain the origin of the vast energy output observed to liberate from quasars.

Since then the BH-proposal has been repeatedly adopted and their mass-regime has been continuously expanded to finally span the whole mass spectrum, ranging from the microgram scale up to billion solar masses.

Although BHs are unobservables by definition, the dynamical behavior of matter and stars in their vicinity in principle should disclose the properties of these objects. In particular, the distinguished depth of their gravitational well, enables BHs to convert a significant portion of the potential energy of matter or objects approaching the BH into other forms of energy, such as thermal or magnetic energy. In fact there are additional observational techniques that are used for predicting the depth of the gravitational well of BHs, e.g., the iron $K\ \alpha$ emission lines associated with rotating plasmas in accretion disks, the Lorentz factor of ejected plasmas from their vicinity and possibly through gravitational waves detectors to be employed in the near future (see Mueller 2007, for additional detection methods).

Most BHs in binaries are considered to have masses of the order of $10\,\mathrm{M}_\odot$. The massive ones are generally found at the center of galaxies with masses ranges between several million up to several billions solar masses. In most cases they are observed to be radiatively active, implying therefore that they are accreting from or ejecting matter into their surrounding.

Nevertheless, these techniques hint to the existence of objects that differ from normal or compact stars, such as old stars, white dwarfs and neutron stars; however they definitely are unable to determine their nature.

Difficulties with the classical BH-Proposal

Consider an object of Mass M and spin=0 embedded in vacuum (see Fig.1), how does the spacetime around this object look like?
From the point of view of GR, the equations to be solved are $\mathrm{G}_{\mu\nu} = 0$.
The solution to this problem is a metric, which was forwarded to Einstein by Schwarzschild just two months after the former published his theory of GR. The metric reads:

$$\mathrm{ds}^2 = \mathrm{c}^2(1 - \mathrm{r_s/r})\mathrm{dt}^2 - \frac{\mathrm{dr}^2}{1 - \mathrm{r_s/r}} - \mathrm{d}\bar{\Omega}^2, \qquad (2)$$

where ds, c, t, $\mathrm{r_s}$ correspond to the proper distance between two events in this spacetime, the speed of light, the time measured by a fixed observer at infinity and the Schwarzschild radius $r_s = 2GM/c^2$, respectively. The term $\mathrm{d}\bar{\Omega}^2 = \mathrm{r}^2\mathrm{d}\Omega^2 = \mathrm{r}^2(\mathrm{d}\theta + \sin^2\theta\ \mathrm{d}\varphi^2)$ correspond to a differential area on the surface of a sphere of radius r.

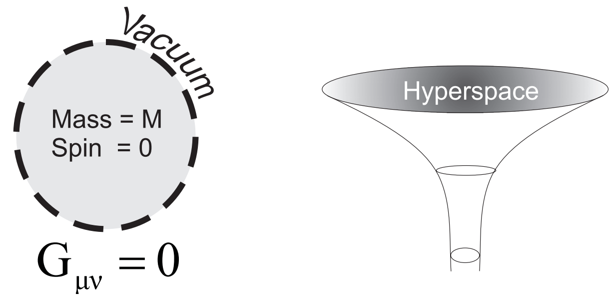

Figure 1. The gravitational field of a non-rotating object of mass M embedded in vacuum is equivalent to a curved spacetime having the event horizon as a boundary. The hyperspace on the right is a projection of the four-dimensional curved spacetime.

As it will be shown below, the Schwartzschild's proposal raises various fundamental questions in GR rather than providing a solution to a problem:

- The Schwarzschild solution contains two singularities: r=0 and $\mathrm{r} = \mathrm{r_s}$. While the former is intrinsic and unremovable, the latter one can be removed by appropriate coordinate transformation, e.g. using the Eddington-Finkelstein coordinates (Hobson et al. 2006). The contraction of the Riemann curvature tensor at the event horizon: $R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau}$ yields a well-defined finite number, which implies that the curvature at this radius is well-behaved and that a co-moving observer will feel nothing special as she/he crosses the even horizon.
  In fact it was shown by Penrose (1965) and Hawking (1967) that if Einstein general theory of relativity is correct and the stress-energy tensor satisfies certain positive-definite inqualties, then spacetime singularities are inevitable. This may imply that all quantum information generally associated with the matter-building the BH will be completely destroyed.

  Moreover, as the lightcones tend to close up near the event horizon, $r = r_s$, our connection to observers approaching the event horizon will be continuously weaker and will be completely lost inside $r = r_s$, which implies that we will never know if this observer ever crossed the horizon.

  This raises a serious question about the progenitor of BHs. If these were massive stars that run out of energy generation via nuclear fusion at their centers followed by a dynamical self-collapse under their own self-gravity, then from our point of view as distant observers they must be still collapsing and would cross the event horizon with the speed of light after infinite time. Equivalently, the progenitors of BHs are massive stars that collapsed into rings of matter that surround the event horizon and whose particles approaching the horizon, but which will never reach or cross it.

- A photon of energy $E_0 = h\nu_0$ measured in our frame will be infinitely blue-shifted at the event horizon. Assuming global energy conservation, this indefinite gain of energy must be extracted from

the field and therefore would cause a non-negligible perturbation to the spacetime. As a consequence, each photon that approaches the horizon will cause a measurable change of spacetime curvature around the BH, which mathematically means, that the solution may not be stable against external perturbations.

- The entropy problem of BHs has been investigated by Bekenstein (1973), who found that the entropy of a BH scales as the number of Planck spheres accommodate-able within the 2D projection area of a BH, or equivalently:

$$S \sim k_B (\frac{r_S}{\ell_P})^2 \approx k_B \times 10^{77} \, (\frac{M}{M_\odot})^2. \tag{3}$$

k_B, and ℓ_P correspond to the Boltzmann constant and the Planck length, respectively.
Assuming a true association of the thermodynamical variables with the BH thermodynamics, then the entropy of progenitor appears to increase by at least twenty orders of magnitude during a transition into a BH phase. This may imply that a large amount of information is hidden behind the horizon and that a deep holographical connection between the horizon as a surface and to the BH as bulk is at work (Padmanabhan 2006)

- The information paradox repeatedly discussed by theoreticians has not been satisfactorily solved yet.

Noting that the phenomena of particle-wave duality in quantum theory would enable particles to cross the event horizon as waves, then the information about the pre-collapsed state of matter must be still found in the non-vanishing wave-tail. Such extraction of information is possible due to the unitarity requirement of the Hamiltonian operator describing the quantum state of the infalling particles. However, the only retrievable informations from BHs is Hawking-quanta, which is of black-body type and therefore featureless (s. Fig. 2).

Similar to the emitted black body radiation from the sun's photosphere, these photons transmit highly diffused information about their paths inside the Sun due to their random-walk motions in optically thick media. Thus the Hawking radiation carry information about the physical conditions at or outside the interface, where they make abrupt transition from the opaque spacetime domain inside the event horizon into the optically thin and roughly flat spacetime outside it (s. Preskill 1992, for further details).

Also, it is not clear, whether energy stats of matter and the behavior of atoms in the vicinity of the event horizon would obey a Maxwell-Boltzmann like-distribution and whether the Planck function would continue to properly describe the photon spectrum.

- The Schwarzschild solution (Eq. 2) unequivocally shows, that GR is capable of singling out two events in the spacetime and declare them to be singular in a deterministic manner. However, this approach is in complete contradiction to the fundamental principles of quantum field theory and in particular to the Heisenberg uncertainty principle (Fig. 2).

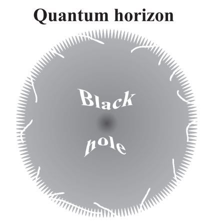

Figure 2. The modern picture of a quantum horizon governed by quantum fluctuations and quantum tunneling, giving rise to the emission of quanta. The semi-classical approach of Hawking predicted that a BH of mass M emits BB-radiation at temperature $T_{BB} = 1/(8\pi G M)$.

- While the progenitors of stellar black hole are well-understood, the evolution of supermassive black holes (SMBHs) is still a controversial issue. It is believed that the Pop III stars in the early universe must have been sufficiently massive, as the corresponding Jeans mass was of order $10^3 M_\odot$ (Bromm 1999).

Such metal-free massive stars must have formed at cosmological redshifts between $8 \leq z \leq 20$ and should have collapsed relatively fast to form the first massive black holes. Since then their mass must have grown exponentially through repeated mergers and accretion of matter to acquire typical quasar masses of the order $10^9 M_\odot$. However, the cosmological simulations were able to show certain condensations of clouds but not the final mass that exclusively go to form the massive star. In fact such modeling is associated with various numerical difficulties that severely limit the reliability of such large scale simulations.

Furthermore, the mass spectrum of BHs suffers of a gab which ranges from 100 to 10000 $M_\odot$. The origin of this gab is unclear, especially because this range of masses could have been conveniently covered by the collapse of Pop III type stars.

2. THE LONG WAY TOWARD BH-ALTERNATIVES

During the last decade several alternatives to BHs have been proposed. A considerable attention was given to dark energy stars (henceforth DESs) and gravitational vacuum stars (henceforth Gravastars) that have been proposed by Chapline et al. (2001); Mazur & Motolla (2004).

Inspired from the $\Lambda-$cold dark matter cosmology (ΛCDM; Peacock 2011)), in which the vacuum energy is considered to be responsible for the accelerating expansion of the universe, one may replace the inner-horizon region of a BH by a vacuum-like core or a De Sitter spacetime.
In this case the effect of gravity is a repulsive rather than impulsive. As in the case of dark energy in cosmology, the energy density in the core is set to be constant whereas the pressure is equal to the negative energy, i.e., $P = -\rho$.

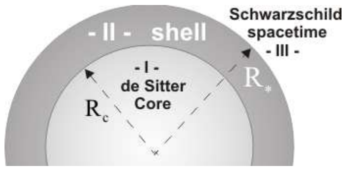

Figure 3. The gravastar model consists of three sub-domains: a central de Sitter core, a Schwarzschild empty space outside the object and a thin spherical shell in-between filled with normal matter.

Such a negative pressure is a typical phenomena on the scale of quantum fluctuations as the Casimir-effect shows. Furthermore, the equation of state (EoS), $P = -\rho$, is an inevitable consequence of Einstein's field equations describing an isotropic and homogeneous universe. Using the Robertson-Walker metric, the Friedmann equations yield the following evolutionary equation for the scaling factor, "a":

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \tag{4}$$

where Λ denote the cosmological constant and $\ddot{a}$ corresponds to the second time derivative of "a" (see Peacock 2011, for further details).

As revealed by cosmological observations, including WMAP and high-redshift Type Ia supernovae (Komatsu et al. 2011), $\ddot{a} > 0$, which implies that the RHS of Eq. (4) must be positive. However, if vacuum energy density is due to zero-point energy of quantized fields, then Λ must be a Lorentz invariant and therefore can be traced back in the history to make it negligibly small compared to the other terms in the equation. Consequently, the term $\rho + 3P$ must be negative, hence $P < -\rho/3$, or generally, $P = \omega\rho$, where $\omega < -1/3$. In fact, recent WMAP cosmological observations reveal a very narrow range for $\omega : -1.24 \leq \omega \leq -0.86$ (Komatsu et al. 2011), implying therefore that $P = -\rho$ as EoS for the vacuum core can be safely used.

In the case of a Gravastar ((see e.g. Mazur & Motolla 2004)), the object consists of the following main three domains (s. Fig. 3):

1. Region I: $0 \leq r \leq R_C$, where R_C is the core radius. This region is governed by de Sitter spacetime governed by the EoS $P = -\rho$.
2. Region III: $r > R_\star$, where $R_\star$ is the radius of the object. In this domain $\mathrm{T}^{(\mathrm{matter})}_{\mu\nu}$ and Λ are set to vanish completely, hence the EoS reads: $P = \rho = 0$.
3. Region II: $R_C < r < R_\star$ corresponds to a spherical shell filled with normal matter that obeys the EoS: $P = \rho$.

The aim is then to search for a global solution to the field equations:

$$G_{\mu\nu} = \kappa G_{\mu\nu} + g_{\mu\nu}\Lambda, \tag{5}$$

that fulfills the boundary conditions across the different interfaces separating the three regions. The solution procedure relies on proposing the following metric solution:

$$\mathrm{ds}^2 = \mathrm{f(r)dt}^2 - \frac{\mathrm{dr}^2}{\mathrm{h(r)}} - \mathrm{d}\bar{\Omega}^2, \tag{6}$$

where $f(r)$, and $h(r)$ are metric coefficients to be found. Unlike Newtonian physics in Eucliedian geometry, the distribution of energy in each region may have a significant impact on the curvature of spacetime. Hence a GR-consistent matching procedure, such as the Israel junction condition Israel (1966) is required in order to construct a global solution that satisfies the conditions in the three different domains.

One possible solution runs as follows:

- In the core region, $f(r) = C\,h(r) = C\,(1 - H_0^2 r^2)$, which is similar to the metric coefficients in the case of de Sitter spacetime in cosmology. The coefficient H_0 is an integration constant that is analogous to the cosmological Hubble constant, though it has completely different value.
- In the outermost region, the solution is identical to that of Schwarzschild, i.e.,
$$f(r) = h(r) = 1 - \frac{r}{r_s}.$$
- In the intermediate region and in the limit $R_C \longrightarrow R_\star$, the following analytical solution was obtained:
$$h(r) = 1 - \frac{2Gm(r)}{r\,c^2} \simeq \epsilon,$$
where $m(r)$ is a continuous mass function, so that $dm(r) = 4\pi\rho r^2 dr$. ϵ is an integration constant of order the Planck mass M_P divided by the mass of the object. Most importantly, the integration constant ϵ is strictly positive, which implies that r_s is less than $r_\star$, therefore prohibiting the formation of an event horizon.
Following Mazur & Motolla (2004), the thickness of the shell was estimated to be of order: $\ell \sim \sqrt{\ell_P r_s} \approx 10^{-14}$ cm, where ℓ_P is the Planck length. Thus, for a one solar mass gravastar , ℓ would hardly exceed the radius of a proton.

Thus the shell is actually an extremely thin membrane rather than a normal matter-contained shell. Moreover, the model requires a conversion mechanism of unknown nature to operate efficiently at the base of the membrane, whose function is to enforce normal matter to undergo a phase transition into a de Sitter vacuum state.

The reliability of the gravitational Bose-Einstein condensates as BH-alternatives

Similar to cosmology, vacuum-dominated cores must expand. The effect of the membrane is then to limit/decelerate the expansion rate, so to maintain these object in hydrostatic equilibrium. (Ghezzi 2011) showed that an anisotropic pressure is required for ensuring dynamical stability of dark energy objects, though a physical origin was not provided.

On the other hand, all normal astrophysical objects known to date, including the relativistic ones, are found to have compactness parameters that are strictly less

than 1/2 (s. Table 1).
Martin & Visser (2003, see also the references therein) found that under a variety of stellar-structure conditions, the compactness parameter has an upper limit: $\mathcal{C} < 4/9$. Therefore, a one solar mass dark energy object (DEO) with a radius $R_\star = r_s + \ell \approx (1+\sqrt{\epsilon})\, r_s$ would have the compactness parameter $\mathcal{C} = 0.5 - 10^{-14}$, which implies that DEOs, if isolated, are almost indistinguishable from their BH-counterparts.
However, the extraordinary compactness of these objects, while having solid surfaces, rises several fundamental questions about their reliability and viability in nature as elaborated in the following.

	$\mathcal{C} = GM_\star/r_\star c^2$
Sun	$< 10^{-5}$
White dwarfs	< 0.0004
Neutron stars	< 0.16
Quark star*	0.37
Dark energy star*	$\approx \frac{1}{2}(1-\sqrt{\epsilon})$
Schwarzschild BH	$\frac{1}{2}$
Kerr BH	1

*Hypothetical models

Table 1
Different astrophysical objects and their corresponding compactness parameter $\mathcal{C}$.

1. While the gravastar model is based on a fluid approach, the connection to quantum effects in vacuum has been performed through the integration constant ϵ solely, which is rather an ad hoc approach. It is unclear, what are those microscopic quantum effects that would lead to different integration constants for different masses in a universe of well-defined universal constants. A reasonable analysis should ensure a scaling out of the dependence of ϵ on the mass in order to yield a universal constant that applies for a reasonable range of the mass function characterizing astrophysical BHs.

2. Similarly is the proposed formation of gravitational Bose-Einstein condensates -GBECs and superfluidity of their cores. Although globally stable condensates have not been verified experimentally yet, these are expected to form when the constituents are cooled down to a temperature near absolute zero. The particles then congregate into a single macroscopic quantum state. In this picture, the superfluid condensate is analogous to a vacuum state, its excited states to normal matter and its surface to a quantum critical shell. Following (Chapline 2004), when ordinary particles enter the quantum critical shell they morph into heavy Bosonic particles.

   In fact, the long time scale of the post-glitch recovery of the Crab and Vela pulsars is a strong observational evidence that superfluidity might be a natural phase governing the flow dynamics in the cores of relativistic neutron stars. Although the core's temperature in a NS is of order several million degrees, this is still two to three orders of magnitude lower than the dominant electron Fermi or the effective Coulomb temperatures characterizing the core's matter. These conditions are equivalent to a terrestrial superfluid with $T \approx 10^{-3}\, K$. However, superfluidity in cores of pulsars is a favored phase due to presence of neutron-proton two-fluid nature with $n_n/n_p \approx 30$, which gives rise to n-n and p-p pairing. Given the high rotational speed of pulsars, the superfluid ought to consist of discrete array of quantized vortex lines elongated parallel to their rotation axis. The Crab pulsar, for example, is expected to contain $N \approx 5.3 \times 10^{18}$ vortex lines. Due to the superconductivity of the core's matter, those vortex lines that are coupled to the magnetic field must migrate randomly outwards, leading to the loss of radiation reaction torques and subsequently decelerating the core's rotational speed (Alpar & Sauls 1988).

When extending this scenario to SMBECs several difficulties may emerge, that could potentially prohibit their formation.

Consider for example the time-evolution of the pressure, P(t), at the center of a neutron star. While the dominant contribution is due to the non-thermal degenerate pressure, these objects have still to spend several million years in order to liberate the vast thermal energy trapped in the core during the dynamical collapse of the progenitor. A newly born neutron star is expected to have a central temperature of $T_C \approx 10^{11} - 10^{12}$ K, but which decreases quickly thereafter to reach several million degrees through extensive neutrino emission. Without extensive neutrino emission, the stored thermal energy in the core would alter the force balance and may lead to completely different evolutionary tracks. In particular, the superfluidity would diminish.

As the progenitors of a stellar mass Bose-Einstein condensate (BECs) must be much more massive than that of a NS, the total thermal energy trapped in the core is of order $\mathcal{U} \approx V_s^2\, M_{BEC}$, where V_s is the sound speed, assuming isothermal core. As this thermal energy would neither disappear in a singularity, as the case in BHs, nor being expelled to the surrounding regions[2], $\mathcal{U}$ must eventually be comparable to the rest energy of the core. Thus V_s is relativistic and therefore is much larger than the critical speed, beyond which superfluidity will be destroyed. Furthermore, as the crust is made of normal matter, it would serve as source for electromagnetic radiation. This loss of energy would cause the core to shrink and subsequently collapse into a BH.
We note that in the absence of self-gravity, it was experimentally verified that external MFs tend to shrink the condensates, enhance the self-interactions and reconnection of vortices and subsequently lead to their collapse or explosion as

[2] Due to the extremely large gravitational redshift

bosonova (Wieman et al. 2001).

In the case that the BEC-phase is reached via a quasi-stationary contraction of a normal matter core, then the heat capacity of the matter in the crust must have been continuously decreasing to reach a small, but still a positive critical value, below which the BEC would cease to thermally interact with the surrounding media. Only a negligibly small fraction of $\xi = \sqrt{1-2\mathcal{C}}$ of the total internal energy would find its way outwards, while the rest is being trapped in the core or diffuses backwards from the crust into the core.

To conclude: the enormous thermal energy trapped in the core during the collapse of the progenitor in combination with magnetic fields and large conversion efficiency of kinetic into thermal energy via shocks at the surface would eventually act to suppress the superfluidity and superconductivity of the core and lead to a reverse phase transition.

A further uncertainty of SMBECs is the causality problem. Consider for example the SMBH powering the high red-shifted quasar PKS 1020-103, whose mass estimated to be $M \approx 2.62 \times 10^9 M_\odot$ and accretion rate of several solar masses per year. The light crossing time of this object is $\tau_{LCT} \sim 2\,r_s/c \approx 7\ hours$, which is the shortest possible time scale required for a global re-adjustment of the core to external perturbations.

Let us consider the case, in which the core consists of a single bosonic condensate with a specific macroscopic quantum state. Unlike white dwarfs and neutron stars, in which the Pauli exclusion principle prevent their collapse, in the case of boson-like condensates it is the Heisenberg uncertainty principle that apposes self-collapse. The zero-energy state would allow the density of bosonic objects to be much larger than in fermionic objects. The critical mass of bosonic objects most likely obeys the correlation: $M_{CB} \sim M_P^2/m_B$, and $M_{CF} \sim M_P^2/m_F^2$ for fermionic objects. m_B and m_F here denote the mass of an individual bosonic and fermionic particles, respectively. Provided that m_B is much less than the proton mass, boson objects in principle could be much more massive than their fermionic counterparts.

If a massive boson condensate consists of one single vortex line with radius r_s and rotational frequency Ω_{BEC}, then it is easy to verify from the integral of the quantized circulation:

$$\oint V \cdot dl = \frac{h}{2m} n,$$

that: $$\Omega_{BEC} = 6 \times 10^{-72} \left(\frac{M_\odot}{M_{BEC}}\right)^3. \tag{7}$$

Here "V, l, h, m, n" correspond to the circulation velocity, path around the vortex, Planck constant, the mass involved in the vortex core and the number of vortices, respectively.

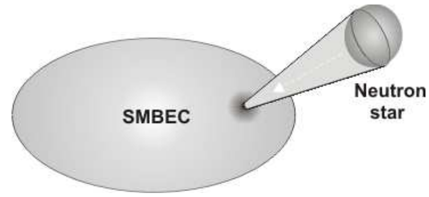

Figure 4. A frontal crash of a neutron star with a non-rotating supermassive Bose-Einstein condensate.

Unless SMBEC are born missive, their cosmological growth will normally be associated with an increase of the rotational energy, which enforces the cores to rotate with a speed much higher than Ω_{BEC}.

Let us now consider the case, in which the core of the above-mentioned quasar is being hit by a neutron star. Noting that the mean surface density of the quasar is: $<\Sigma>_{BEC} = M/(4\pi\, r_s^2) \approx 10^{11}\, g\, cm^{-2}$, then the ratio of the surface energy density of the NS to that of the SMBEC at the time of the crash is:

$$\frac{\mathrm{E}^{\Sigma}_{\mathrm{NS}}}{\mathrm{E}^{\Sigma}_{\mathrm{BEC}}} \approx 10^9. \tag{8}$$

In evaluating the last expression we have assumed $M_{NS} = M_\odot$ and $R_{NS} = 10^6\, cm$. On the other hand, for a gravastar, for example, the total mass of the membrane is roughly equal to the Planck mass. Thus the mass of the portion of the membrane possibly involved directly in this crash can be estimated to be: $\sim M_{PL}\,(S_{NS}/S_{SMBEC}) \approx 10^{-21}\, g$, where $S_\square$ means the surface of the corresponding object. Thus, the local energy available for resisting the NS-SMBEC crash is of the order of several ergs. Also, the local time scale characterizing the crash event is $\tau_{LOC} \sim R_{NS}/c \approx 10^{-9} \tau_{GD}$, where τ_{GD} is the global dynamical time scale of the SMBEC.
Given that the input of energy through the crash exceeds the locally available one by several orders of magnitude and that the NS would crash into membrane almost with the speed of light, we conclude that a considerable portion of the membrane and the enclosed portion of the SMBEC would be completely destroyed early enough before the condensate could react dynamically to maintain global stability. In the case of stellar mass BECs, the input of energy associated with such a crash would destroy the whole condensate.

3. When feeding a gravitational BEC with the mass rate: $\dot{M} = c^3/2G$, then the horizon would grow with the speed of light. This is equivalent to inject the core with one solar mass per 10^{-5} seconds, which is much longer than the duration of a NS-SBEC crash-event, which is expected to be of order $\epsilon\,(r_s/c)$. Let us assume that the membrane of a BEC be located an epsilon small outside the event horizon, i.e. at $r_{BEC} = (1+\epsilon)\, r_s$ and that this configuration is applicable to DEOs and independent of their mass, then it is easy to verify from

Figure 5. The Lorentz factor versus the compactness parameter $\mathcal{C}$ is plotted for three different jet launching parameters: $\alpha_0^2 = 1, 0.9, 0.75$ denoted with the black, blue and red colors, respectively.

the mass-radius relation that:

$$\frac{dr}{dt}|_{BEC} = \frac{2G}{c^2}(1+\epsilon)\frac{dM}{dt} > \frac{dr}{dt}|_{BH}. \qquad (9)$$

Consequently, in order to ensure that the surface of the newly formed core still lays outside the event horizon, its surface must contradictory grow with a superluminal speed.

4. The settling normal matter from the surrounding would create a multi-component fluid in the crust, establishing herewith the appropriate conditions for the creation of different Fermi surfaces for fermions and bosons and therefore applying a magnetic tension, which would tend to couple the core to the crust dynamically. As a consequence, the core break into a multiple number of vortex lines threaded by magnetic fields. Similar to the superfluid and superconducting cores in NSs, the vortex lines in the core of a SMBEC that are coupled to the magnetic field would migrate from inside-to-outside, enhancing thereby self-interaction and vortex reconnection and diminishing thereby the superfluidity of the core. In this case, we anticipate the surface of a SMBEC to be magnetically active with intense eruption events, though hardly observable. Similar to cores in NSs, the core-crust coupling through magnetic fields would cause SMBEC to decelerate its rotational speed.

   Moreover, we note that rotating superfluids in BEC-cores most likely are similar to rotating superfluids between concentric spheres. The latter was verified to be unstable against non-axisymmetric perturbations compared to normal rotating Couette flows (Barenohi & Jones 1987).

5. Highly collimated relativistic jets have been observed to emanate from the vicinity of accreting BHs and NSs. The Lorentz factors characterizing the propagational speed of these jets were verified to correlate with the compactness parameter of accreting objects (see Hujeirat et al. 2002, 2003, and the references therein). To first order approximation, we may assume jet-velocities to linearly correlate with the escape velocity, i.e., $V_J = \alpha_0 V_{es}$, whereat α_0 is a constant coefficient of order unity. The Lorentz factor can be expressed then as function of the compactness parameter as follows: $\Gamma = 1/\sqrt{(}1 - \alpha_0^2\mathcal{C})$. Figure (5) shows that Γ is hughly sensitive to α_0, indicating therefore that large $\Gamma-$factors between 10 to 20 may be obtained if the jet velocity is a significant fraction of the escape velocity. Equivalently, the jet-plasma must star its outwards-oriented motion from the very vicinity of the object's surface or from nearly the event horizon.

   We note that the formation and acceleration of jet-plasmas around accreting relativistic objects is considered to be the outcome of complicated coupling processes between the accretion disk, the central object and the jet-plasma, whereby magnetic fields play a crucial role (Hujeirat 2004, 2011). Jets emanating from around surface-free objects are generally observed to be radio-dominated and their plasmas propagate with larger Lorentz factors compared to those of neutron stars. Therefore, in the case of accreting DEOs the solid surface in combination with magnetic fields threading the normal-matter-made membrane would give rise to an equally significant thermal and radio emission with strong variabilities. These could serve as guide to fix the compactness parameter of the object. However, a stellar type DEO would display variabilities whose maximum frequency behaves as:

$$\nu_{QPO} \approx \frac{\sqrt{\mathcal{C}}}{2\pi}\frac{c}{r}.$$

   If the microquasar GRS 1915+ 105 were a DEO, for example, then it would display quasi-periodic oscillation around 400 Hz. This is approximately one order of magnitude larger than the revealed value by observations. Moreover, the dominant radio over thermal emission characterizing this objects rules out the possibility of a solid surface.

6. The intensity of the magnetic fields must amplify to reach roughly equipartition values as the accreted plasma shocks the crust of the SMBEC (Fig. 6). Consequently, a boundary layer must form, whose thickness, ℓ, scales as:

$$\frac{\ell}{r_\star} \sim \mathcal{C}(\frac{V_A}{c})^2.$$

   V_A in this correlation stands for the Alfv̀en speed. Thus, the boundary layer (BL), which is most likely filled with virially hot elementary particles, is effectively a photosphere with a macroscopic thickness rather than a membrane with sub-atomic microscopic width. In this case, large scale magnetic fields are capable of communicating the presence of the solid crust to the surrounding media. The freely falling charged particles would then decelerate in the BL, emitting thereby a considerable amount of their energy at the synchrotron frequency and give rise to a total synchrotron lumi-

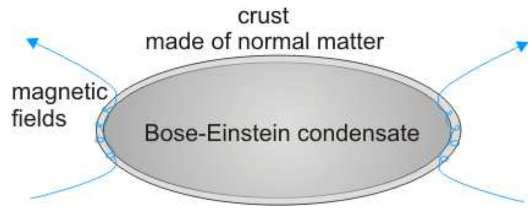

Figure 6. Magnetic fields threading the matter made crust (; unscaled thickness) of the condensate.

nosity of the order:

$$L_{syn} \sim 10^{45}\ n^2\ (\frac{M_9}{T_4})^3\, erg\, s^{-1}.$$

n, T_4, M_9 correspond to the number density, the pre-shock temperature of the accreted plasma and the mass of the SMBEC in units of $10^9\ M_\odot$, respectively.

Consequently, such a large radio luminosity from a geometrically thin BL would be optically thick to synchrotron emission and the BL would appear as a bright photospheric ring that surrounds the SMBEC. However, such rings have not been observed yet, though they would be comfortably observable with today's detectors.

3. SUMMARY

Various aspects of black holes and dark energy objects as BH-candidates have been discussed and the draw backs of both kind of objects have been addressed. In particular, it is argued that gravitational Bose-Einstein condensates, as alternatives to supermassive black holes, suffers of a causal problem and should be ruled out on the case of merger events. Moreover, the superfluidity of SMBEC-cores most likely is a short-living phase and would not survive non-axisymmetric perturbations initiated by the magnetic fields, external forces or mergers. Such perturbations would enhance the vortex-line migration phenomena, increase their rate of interaction and reconnection and finally turn the core dissipative, leading subsequently to core collapse into a black hole or explode as bosonova.

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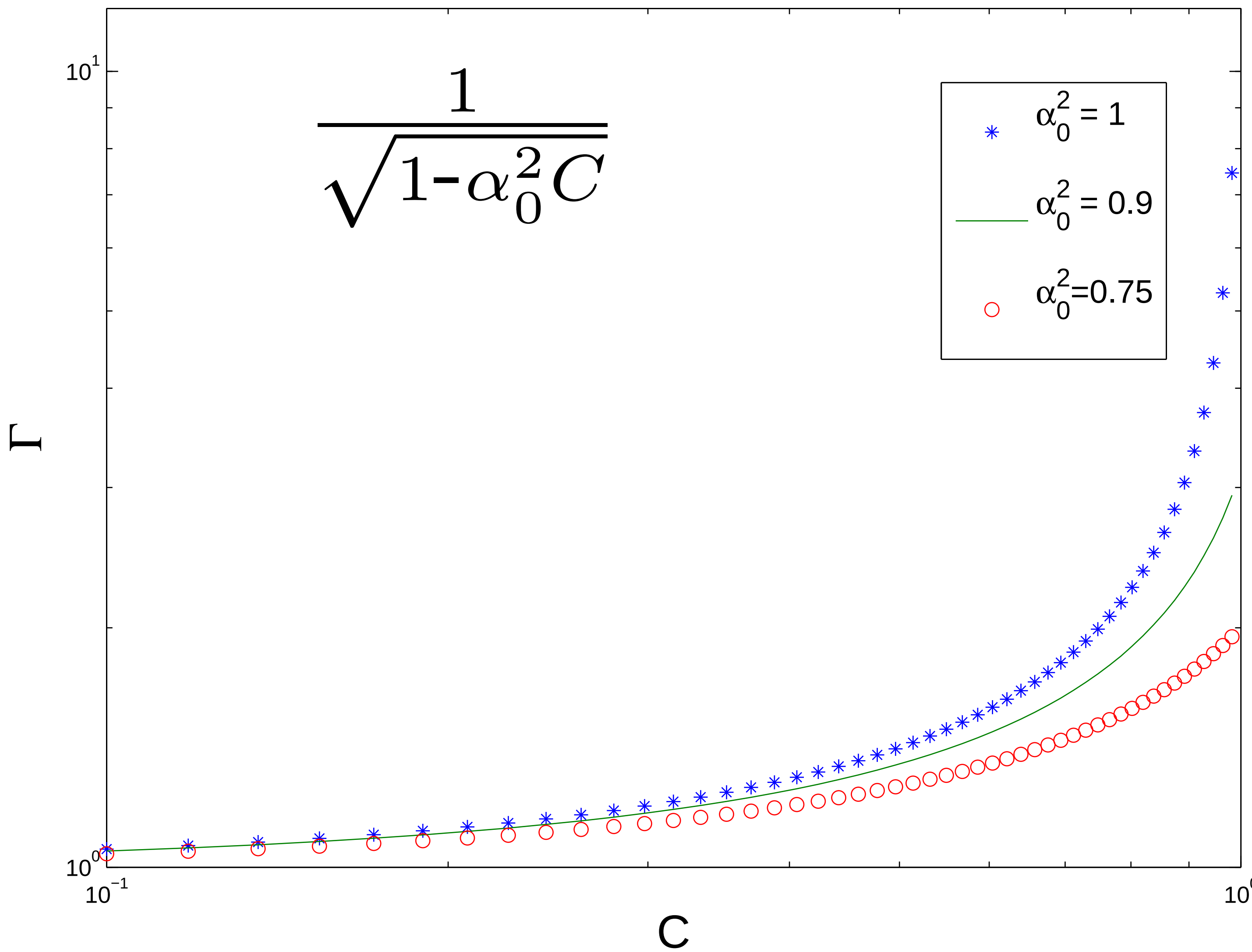

$$\frac{1}{\sqrt{1-\alpha_0^2 C}}$$
$\alpha_0^2 = 1$
$\alpha_0^2 = 0.9$
α_0^2=0.75
Γ
C
10^1
10^0
10^{-1}
10^0